EECE 460: Control System Design
Midterm #1

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This midterm is closed-note, closed book, and only non-graphical calculators are allowed. A two-sided, one-page cheat-sheet is allowed and *must be handed in with your midterm.* For full credit, show all your work.

SOLUTIONS

Student Name Student ID #

Problem #	Actual points	Possible points
1		40
2		30
3		30
Total:		100

1 PID Control (40 points)

Consider the nominal plant model

$$G_0(s) = \frac{10}{(s+1)} e^{-2s}$$

1. (10 points) Using λ-tuning, derive a PI controller

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For $G_0(s) = \frac{k_p}{1+Ts} e^{-sL}$.

PI $C(s) = K \cdot \frac{1+sT_i}{sT_i}$ with

$T_i = T \quad K = \frac{T}{k_p(\lambda+L)}$

Here $k_p = 10$; $T = 1$; $L = 2$

$$\boxed{T_i = 1. \qquad K = \frac{1}{10(\lambda+2)}}$$

e.g. with $\lambda = 0.5$ $\boxed{\begin{array}{l} T_i = 1 \\ K = \frac{1}{25} = 0.04 \end{array}}$

2. (10 points) Still using λ-tuning, now derive a PID controller

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λ-tuning for a PID controller:

$$C(s) = \frac{(1+sL/2)(1+sT)}{K_p s(L+\lambda)}$$

$$\boxed{\begin{aligned} T_i &= T = 1 \\ T_d &= \frac{L}{2} = 1 \\ K &= \frac{1}{K_p}\frac{T}{L+\lambda} = \frac{1}{10(\lambda+2)} \end{aligned}}$$

3. (10 points) Using Haalman's method design a PID controller for the system

$$G(s) = \frac{1}{(1+2s)(1+3s)}e^{-s}$$

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$$G(s)\,C(s) = \frac{2}{3Ls}e^{-sL}$$

$$\frac{C(s)}{(1+2s)(1+3s)}\cancel{e^{-s}} = \frac{2}{3Ls}\cancel{e^{-s}}$$

$$\boxed{C(s) = \frac{(1+2s)(1+3s)}{1.5s}}$$

4. (10 points) For the same system, design a Smith predictor using the previously derived PID controller. Give both a diagram and the equation representing this Smith predictor.

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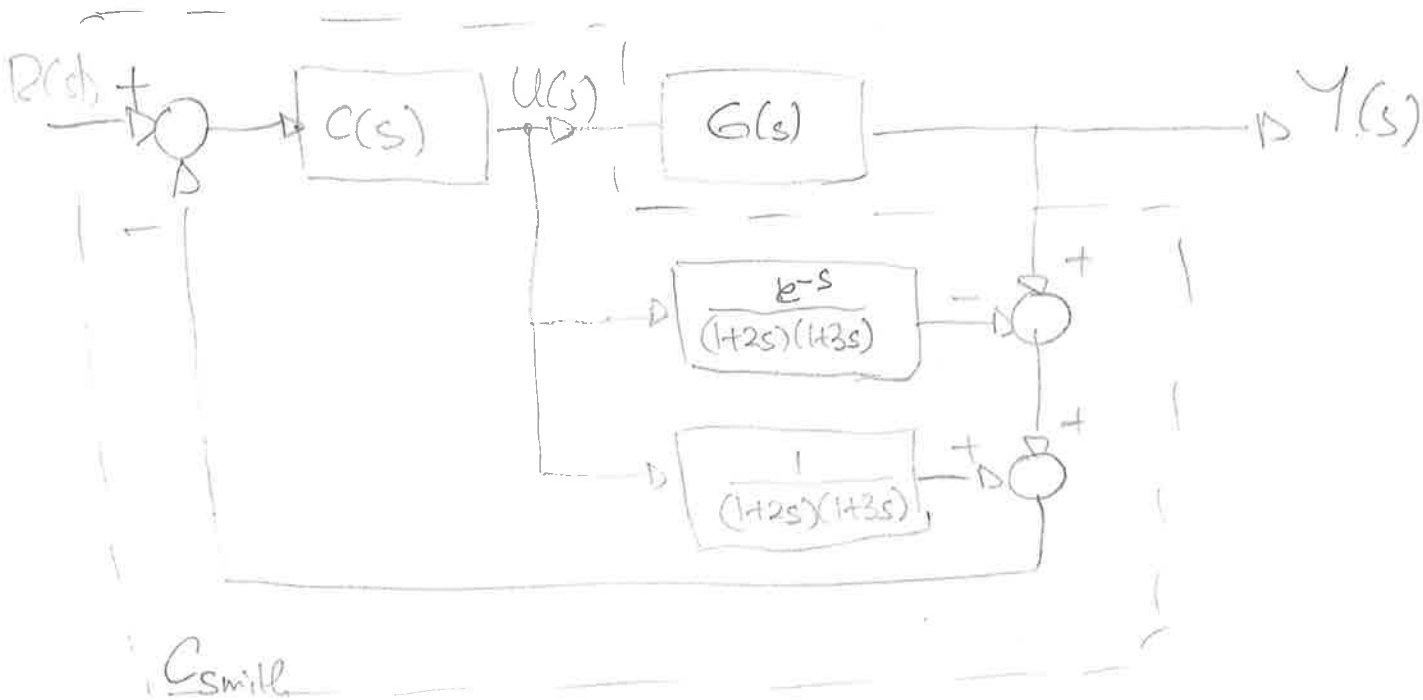
We can use the previous PID controller in this configuration:



with

$$C_{smith} = \frac{C(s)}{1 + \bar{G}C(s)(1 - e^{-Ls})}$$

$$= \frac{(1+2s)(1+3s)/1.5s}{1 + \frac{1}{1.5s}(1 - e^{-s})}$$

$$\boxed{C_{smith} = \frac{(1+2s)(1+3s)}{1.5s + 1 - e^{-s}}}$$

2 Pole Placement (30 points)

Consider again this system:

$$G_0(s) = \frac{4}{(s^2 + s + 4)}$$

1. (15 points) Using pole placement, design a minimum-complexity controller giving a closed-loop system with a bandwidth of 5 rad/s.

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$n=2.$ $\deg A_{cl} = 2n-1 = 3.$

$A_{cl} = (s^2+7s+25)(s+10).$ $\deg P = \deg L = n-1 = 1$

$P(s) = p_1 s + p_0$ $L(s) = s + l_0.$

$AL + BP = A_{cl}.$

$(s^2+s+4)(s+l_0) + 4(p_1 s + p_0) = (s^2+7s+25)(s+10).$

$s^3 + (l_0+1)s^2 + (4+l_0)s + 4l_0 + 4p_1 s + 4p_0 =$

$s^3 + 17s^2 + 95s + 250$

$l_0 = 16$ $4 + l_0 + 4p_1 = 95$ $4p_1 = 75$ $p_1 = \frac{75}{4} = 18.75$

$4l_0 + 4p_0 = 250$ $4p_0 = 186$ $p_0 = 46.5$

$$\boxed{C(s) = \frac{18.75s + 46.5}{s+16}}$$

2. (15 points) For the same system, design a new controller that in addition to meeting the above requirement contains an integrator. Show that the controller is a PID.

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deg $A_{cl} = 4$. Can choose e.g

$$A_{cl} = (s^2+7s+25)(s+10)^2 \quad \text{or}$$

$$P(s) = p_2 s^2 + p_1 s + p_0$$

$$L(s) = s(s+l_0)$$

$$(s^2+s+4)s(s+l_0) + 4(p_2 s^2 + p_1 s + p_0) =$$

$$(s^2+7s+25)(s^2+20s+100).$$

$$A_{cl} = (s^2+7s+25)(s^2+s+4)$$

if we want to cancel process poles

$$P(s) = p(s^2+s+4)$$

$$L(s) = s(s+l_0)$$

$$\cancel{(s^2+s+4)}s(s+l_0) + 4p\,\cancel{(s^2+s+4)} =$$

$$(s^2+7s+25)\cancel{(s^2+s+4)}$$

$$s^2 + l_0 s + 4p = s^2 + 7s + 25$$

$$l_0 = 7. \qquad p = \frac{25}{4} = 6.25$$

$$\boxed{C(s) = \frac{6.25(s^2+s+4)}{s(s+7)}}$$

3 Pole Placement (30 points)

Consider the system

$$G_0(s) = \frac{1}{s}$$

1. (15 points) Using pole placement, design a minimum-complexity controller giving a closed-loop system with a bandwidth of 3 rad/s. Is the result surprising?

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deg $A_{cl} = 2n-1 = 1$. Choose $A_{cl} = s+3$

deg $P =$ deg $L = 0$ $P(s) = p_0$ $L(s) = 1$.

$s + p_0 = s+3$ $\boxed{p_0 = 3}$

This is a proportional controller.
Not surprising as system is pure integrator.

2. (15 points) For the same system, modify the controller to force an integrator. What is the advantage of this modification?

$L(s) = s$ $\quad$ $P(s) = p_1 s + p_0$

$\deg A_d = 2.$

$A_{cl} = (s+3)^2 = s^2 + 6s + 9.$

$s^2 + p_1 s + p_0 = s^2 + 6s + 9$

$p_1 = 6 \quad p_0 = 9.$

$$\boxed{C(s) = \frac{6s+9}{s}}$$

PI controller

the advantage of introducing an integrator is that this controller can reject an input step disturbance. The previous one (proportional) cannot.