EECE 460 Homework 3 Solutions

Problem 7.3. *The set of assignment pole problems is solved using the* ***paq.m*** *MATLAB subroutine. The results are indicated in the table below.*

$B_o(s)$	$A_o(s)$	$A_{cl(s)}$	$C(s)$
1	$(s+1)(s+2)$	$(s^2+4s+9)(s+8)$	$\dfrac{12s+54}{s+9}$
$-s+6$	$(s+4)(s-1)$	$(s+2)^3$	$\dfrac{1.24s+4.16}{s+4.24}$
$s+1$	$(s+7)s^2$	$(s+1)(s+7)(s+4)^3$	$\dfrac{(s+7)(72s+112)}{(s+15)(s+1)}$

In the third case, the choice of $A_{cl}(s)$ forces the cancellation of the plant pole at $s=-7$ and the cancellation of the plant zero at $s=-1$.

Problem 7.4. *To achieve zero steady state we need to force an integrator in the controller. Also, to obtain closed loop modes decaying faster than e^{-3t}, we need to choose $A_{cl}(s)$ with all its roots with real part less than -3. We choose $A_{cl}(s) = (s+4)(s+5)(s+6)(s+7)$. Note that one of the roots was chosen in the same location as a plant pole. This will lead to a cancellation and will simplify the solution of the pole assignment equation. Then $P(s)$ will have a zero at $s=-4$.*

$$(s+4)(s+3)\underbrace{s(s+\alpha_0)}_{L(s)} + 3\underbrace{(s+4)(\beta_1 s+\beta_0)}_{P(s)} = (s+4)(s+5)(s+6)(s+7) \tag{7.7.8}$$

Using the ***paq.m*** *MATLAB subroutine the above equation yields*

$$C(s) = \frac{P(s)}{L(s)} = \frac{(62s+210)(s+4)}{3s(s+15)} \tag{7.7.9}$$

Note that the use ***paq.m*** *requires previous cancellation of common factors, in this case, the factor $(s+4)$ in (7.7.9). We then execute the following MATLAB sequence:*

```
>> [X,Y]=paq([1 3 0],3, poly([-5 -6 -7]));
>> L=conv(X,[1 0]);P=conv(Y,[1 4]);
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Problem 7.8. *We first notice that a minimum degree biproper controller (with integration) requires $A_{cl}(s)$ of degree 4 $(= 2n)$. We thus choose*

$$A_{cl}(s) = (s^2 + 7s + 25)(s + 10)^2 \tag{7.7.17}$$

The choice of the double pole at $s = -10$ is arbitrary but for the requirement that they should generate modes[1] *faster than those*[2] *produced by the factor $s^2 + 7s + 25$.*

The associated Diophantine equation is

$$(s^2 - s - 2)\underbrace{s(s + \alpha_0)}_{L(s)} + (-1)\underbrace{(\beta_2 s^2 + \beta_1 s + \beta_0)}_{P(s)} = (s^2 + 7s + 25)(s + 10)^2 \tag{7.7.18}$$

[1] Those modes are e^{-10t} and te^{-10t}

[2] Those modes are $K_1 e^{-3.5t}\cos(\sqrt{12.75}\,t + K_2)$

We thus obtain $L(s) = s(s + 78)$ and $P(s) = -(295s^2 + 1256s + 2500)$.

Problem 7.10. *A Smith predictor is shown in Figure 7.1. We then need to synthesize a controller considering only the rational part, $\overline{G}_o(s)$, of the nominal model, $G_o(s)$, where*

$$\overline{G}_o(s) = \frac{s + 5}{(s + 1)(s + 3)} \tag{7.7.20}$$

The nominal complementary sensitivity is then

$$T_o(s) = \frac{e^{-0.5s}C(s)\overline{G}_o(s)}{1 + C(s)\overline{G}_o(s)} \tag{7.7.21}$$

If the dominant closed loop poles are $-2 \pm j0.5$, and we require integration, we can build a closed loop polynomial of the form $A_{cl}(s) = (s^2+4s+4.25)(s^2+8s+16)$. Thus

$$(s + 1)(s + 3)\underbrace{s(s + \alpha_0)}_{L(s)} + (s + 5)\underbrace{(\beta_2 s^2 + \beta_1 s + \beta_0)}_{P(s)} = (s^2 + 4s + 4.25)(s^2 + 8s + 16) \tag{7.7.22}$$

The solution of the above equation yields $L(s) = s(s + 4.7687)$ and $P(s) = 3.2313s^2 + 14.0187s + 13.6$.