SOLUTIONS SET 4

Problem 10.1. *The disturbance-generating polynomial, $\Gamma_d(s)$ corresponds the denominator of the Laplace transform for each signal. This can be computed using a software package such as MAPLE.*

a) *The Laplace transform of $d_g(t) = 3 + t$ is as follows:*

$$\mathcal{L}\left[d_g(t)\right] = D_g(s) = \frac{3}{s} + \frac{1}{s^2} = \frac{3s+1}{s^2} \qquad (10.10.1)$$

Thus, the generating polynomial is $\Gamma_d(s) = s^2$.

We have to observe that this result represents the solution for the following differential equation:

$$\frac{d^2 f(t)}{dt^2} = 0 \qquad (10.10.2)$$

Where the general solution $f(t) = C_1 t + C_2$, and the given signal corresponds to the particular case when the initial conditions are $f(0) = 3$ and $\left.\frac{df(t)}{dt}\right|_{t=0} = 1$.

b) *In this case, the signal is $d_g(t) = 2cos(0.1t + \frac{\pi}{7})$, and its Laplace transform is as follows:*

$$\mathcal{L}\left[d_g(t)\right] = D_g(s) = \frac{2s}{s^2 + 0.01} \qquad (10.10.3)$$

The generating polynomial is thus, $\Gamma_d(s) = s^2 + 0.01$.

The corresponding differential equation is as follows:

$$\frac{d^2 f(t)}{dt^2} + 0.01 f(t) = 0 \qquad (10.10.4)$$

Where $f(t) = C_1 cos(0.1t + C_2)$ is the general solution. The given signal is a particular member of this solution family, satisfying $f(0) = 2cos(\frac{\pi}{7})$ and $\left.\frac{df(t)}{dt}\right|_{t=0} = -0.2\sin(\frac{\pi}{7})$.

d) *In this case, the Laplace transform of the signal is as follows:*

$$\mathcal{L}\left[d_g(t)\right] = D_g(s) = \frac{s+0.1}{s^2+0.2s+0.05} \tag{10.10.9}$$

Thus, $\Gamma_d(s) = s^2 + 0.2s + 0.05$.

This implies that the associated differential equation is as follows:

$$\frac{d^2 f(t)}{dt^2} + 0.2\frac{df(t)}{dt} + 0.05 f(t) = 0 \tag{10.10.10}$$

Where $f(t) = C_1 e^{0.1t} cos(0.2t + C_2)$ *and the given signal is the solution that satisfies* $f(0) = 1$ *and* $\left.\frac{df(t)}{dt}\right|_{t=0} = -0.1$.

Problem 10.3. *To solve this problem, we note that the controller denominator should contain, as a factor, the generating polynomial* $s(s^2+4)$. *This is necessary and sufficient for the loop to satisfy the IMP requirement. An appropriate synthesis technique is the pole assignment method. Recall that* $(2n-1)$ *is the minimum degree of* A_{cl} *when no IMP constraint is imposed. In our case, we have to satisfy the IMP for* $s = 0$ *(adds one to the minimal degree), and for* $s = \pm j2$ *(adds two to the minimal degree). This leads to a minimal degree for* A_{cl} *equal to* $2n-1+1-2 = 6$.

To simplify the design, we cancel the (stable) poles of the plant with the controller. Thus, we are led to

$$\underbrace{s(s^2+4)(s+l_0)}_{L(s)}\underbrace{s^2+6s+8}_{A_o(s)} + \underbrace{(-s+8)}_{B_o(s)}\underbrace{(s^2+6s+8)(p_2 s^2 + p_1 s + p_0)}_{P(s)} = \underbrace{(s^2+5s+16)(s+8)(s+10)(s^2+6s+8)}_{A_{cl}(s)} \tag{10.10.11}$$

Simplifying the factor (s^2+6s+8) *and solving the equation, we obtain*

$$l_0 = 55.5294 \quad p_0 = 160 \quad p_1 = 78.235 \quad p_2 = 32.5294 \tag{10.10.12}$$

The resulting controller therefore has the following transfer function:

$$C(s) = \frac{32.5294s^4 + 273.41s^3 + 889.645s^2 + 1585.9s + 1280}{s(s^2+4)(s+55.5294)} \tag{10.10.13}$$

Problem 10.4. *For subsequent reference we will describe the plant as* $G_o(s) = \frac{e^{-0.7s}}{(s+1)^2} = e^{-0.7s}\bar{G}_o(s)$

10.4.1 *The general control architecture is shown in Figure 10.1. For this case*

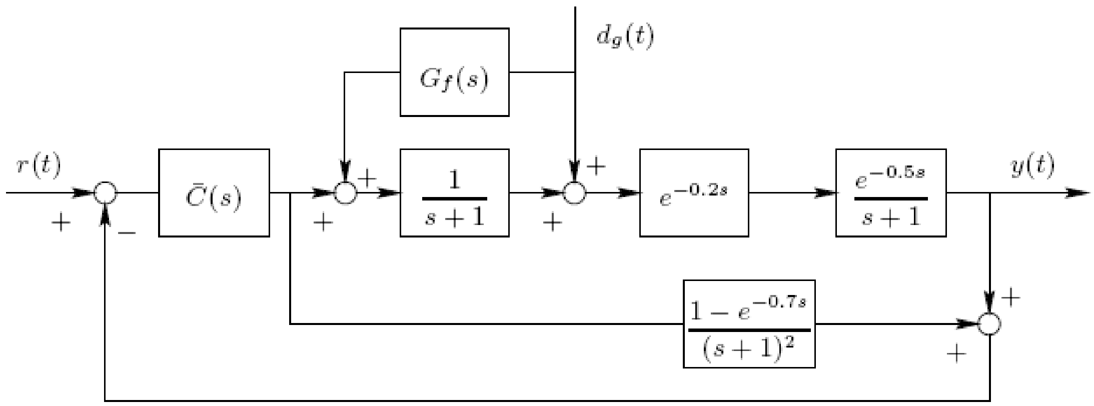

Figure 10.1. First and third degree of freedom control using Smith controller.

we will use a Smith Controller, as suggested. We also recall that $T_o(s) = G_o(s)\bar{C}(s)/(1+\bar{G}_o(s)\bar{C}(s))$.

This leads to the choice of $\bar{C}(s)$ *as*

$$\bar{C}(s) = \frac{4(s+1)^2}{s(s+3)} \qquad (10.10.14)$$

We can now design the third-degree-of-freedom, i.e., disturbance feedforward. To do that we have to build an approximation to $-\left[G_{o1}(s)\right]^{-1}$, *we thus choose*

$$G_f(s) = \frac{K(s+1)}{\beta s+1} \qquad (10.10.15)$$

Where $K=-1$ *and* $\beta = 0.1$. *The choice of* β *is arbitrary, provided that it is much smaller than 1, to achieve a good inverse approximation.*

10.4.2 *When we use cascade control (in conjunction with the Smith controller), we have the control architecture shown in Figure 10.2.*

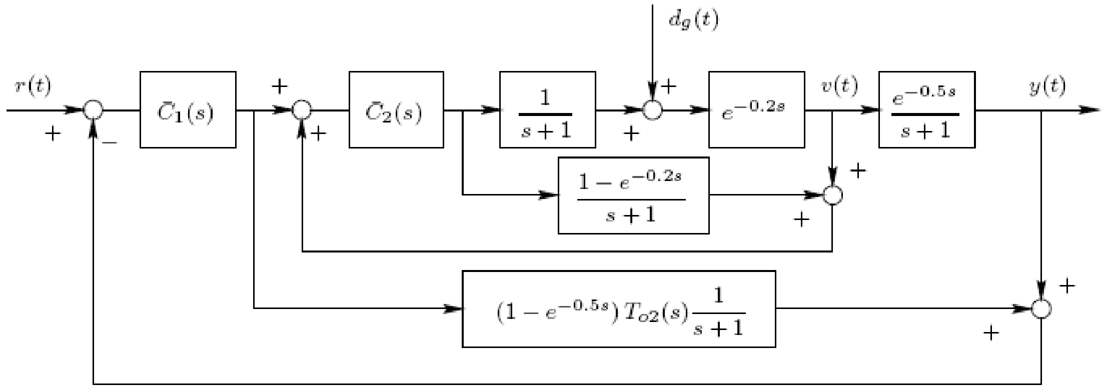

Figure 10.2. Cascade control using Smith controller

Thus, to achieve the desired complementary sensitivity we have an infinite number of alternatives for $\bar{C}_1(s)$ and $\bar{C}_1(s)$. However, we also need to achieve disturbance rejection similar to that obtained in 10.4.2. Since previously we used disturbance feedforward, the effect of the disturbance in the loop output could be made almost negligible. In the alternative cascade architecture, we can only get near to the above ideal result if we design the inner loop to have a very high bandwidth. We thus choose

$$T_{o2}(s) = \frac{100e^{-0.2s}}{s+100} \tag{10.10.16}$$

which leads to

$$\bar{C}_2(s) = \frac{100(s+1)}{s} \tag{10.10.17}$$

Thus

$$\frac{e^{-0.5s}}{s+1}T_{o2}(s) = \frac{100e^{-0.7s}}{(s+1)(s+100)} \tag{10.10.18}$$

From where we can compute $\bar{C}_1(s)$ *to achieve the prescribed sensitivity*

$$\bar{C}_1(s) = 0.04\frac{(s+1)(s+100)}{s(s+3)} \Longrightarrow T_o(s) = \frac{4}{s^2+3s+4} \tag{10.10.19}$$

Note that if had chosen $T_{o2}(s)$ *of relative degree larger than 1, we would have been unable to achieve the desired sensitivity with a proper controller* $\bar{C}_1(s)$. *We leave the details to the reader.*

Problem 10.8. *To achieve goals (i) and (ii) we need that the IMP be satisfied for* $s = 0$ *and* $s = \pm 0.25$. *This requires that the controller denominator has a factor* $s(s^2+0.0625)$. *Using the pole placement synthesis method, we have that the closed*

loop polynomial minimum degree should be 6. If we cancel the plant poles with the zeros of the controller, the Diophantine equation reduces to

$$s(s^2+0.0625)\underbrace{(s+l_0)(s+1)(s+4)}_{L(s)} + (-s+4)\underbrace{(s+1)(s+4)(p_2s^2+p_1s+p_0)}_{P(s)} = (s^2+5s+16)(s+8)(s+10)(s+1)(s+4) \tag{10.10.34}$$

Simplifying this equation, we obtain

$$s(s^2+0.0625)(s+l_0) + (-s+4)(p_2s^2+p_1s+p_0) = (s^2+5s+16)(s+8)(s+10) \tag{10.10.35}$$

which leads to the final controller as

$$C(s) = \frac{109s^4+794.8s^3+2006s^2+2600s+1280}{s^4+132s^3+0.0625s^2+8.248s} \tag{10.10.36}$$