Chapter 15

SOLUTIONS

Problem 15.1. *The general formula to achieve zero steady state error is (see Lemma 15.2, in the book)*

$$Q(s) = s\bar{Q}(s) + [G_o(0)]^{-1} \tag{15.15.1}$$

where $s\bar{Q}(s)$ is any stable rational function.

When we apply this expression to the particular case we have that $G_o(0) = 3/5$. Hence

$$Q(s) = \frac{5}{3} + s\bar{Q}(s) \tag{15.15.2}$$

Problem 15.2. *We have that $T_o(s) = Q(s)G_o(s)$, thus a possible choice for $Q(s)$ is*

$$Q(s) = K\frac{(s+5)(s+10)}{(s+2)+(1.5)^2} \tag{15.15.3}$$

where K is chosen to achieve zero steady state error at $\omega = 0$, i.e. to have $T_o(0) = 1$. This requires $K = 13/60$.

Problem 15.3. *We recall (see Lemma 15.3, in the book) that to achieve zero steady state errors at $\omega = \omega_o$ it is necessary that $Q(s)$ satisfies*

$$Q(s) = \frac{(s^2+\omega_o^2)N_1(s) + N_2(s)}{D_Q(s)} \qquad \text{with} \quad N_2(j\omega_o) = D_Q(j\omega_o)[G_o(j\omega_o)]^{-1} \tag{15.15.4}$$

In this case, $G_o(j\omega_o) = 0.48 - j0.64$. A particular and convenient choice of $N_2(s)$ is such that (see equation (15.3.24))

$$Q(s) = (s^2+\omega_o^2)\bar{Q}(s) + \frac{1}{2j\omega_o}\left([G_o(j\omega_o)]^{-1} - G_o(-j\omega_o)]^{-1}\right) \tag{15.15.5}$$

$$= (s^2+\omega_o^2)\bar{Q}(s) + \frac{s}{\omega_o}\Im\{[G_o(j\omega_o)]^{-1}\} + \Re\{[G_o(j\omega_o)]^{-1}\} \tag{15.15.6}$$

where $\bar{Q}(s)$ is any stable transfer function.

Given that $[G_o(j0.5)]^{-1} = 0.75 + j$, we have that $\Im\{[G_o(j0.5)]^{-1} = 1$, hence

$$Q(s) = (s^2 + 0.25)\bar{Q}(s) + (2s + 0.75) \tag{15.15.7}$$

Note that $Q(\pm j\omega_o)G(\pm j\omega_o) = 1$, i.e. $T_o(\pm j\omega_o) = 1$.

Problem 15.4. *We first recall Lemma 15.6 (in the book), where, after canceling $E(s)$, we have that all stabilizing controllers for an unstable plant with transfer function $G_o(s) = B_o(s)/A_o(s)$ can be expressed as*

$$C(s) = \frac{P(s) + Q_u(s)A_o(s)}{L(s) - Q_u(s)B_o(s)} \tag{15.15.8}$$

where $P(s)/L(s)$ is any stabilizing controller, and $Q_u(s)$ is any stable transfer function.

We can easily verify that $P(s) = 2$, $L(s) = 1$ stabilizes the plant, thus all stabilizing controllers for the given unstable plant can be described by

$$C(s) = \frac{2 + (s-1)Q_u(s)}{1 - Q_u(s)} = 1 + \frac{1 + sQ_u(s)}{1 - Q_u(s)} \tag{15.15.9}$$

After some algebra we can reduce the above expression to

$$C(s) = (1 - s) + (s + 1)V(s) \qquad \text{with} \quad V(s) = \frac{1}{1 - Q_u(s)} \tag{15.15.10}$$

Hence, (15.15.10) *describes all stabilizing controllers for the given plant if $V(s)$ is any minimum phase transfer function. Note that $V(s)$ can be chosen to be unstable provided $Q_u(s)$ is stable.*

Problem 15.5. *The regulation design specifications imply that $Q(s)$ should be chosen so as that $Q(0)G_o(0) = T_o(0) = 1$.*

15.5.1 *We choose*

$$Q(s) = K\frac{(s+1)(s+3)}{s^2 + 6s + 16} \tag{15.15.11}$$

To achieve zero steady state in regulation we have to choose $K = 8$. This leads to

$$T_o(s) = \frac{16(-0.1s + 1)}{s^2 + 6s + 16} \tag{15.15.12}$$

15.5.2 *The design is tested using the SIMULINK file* ***nmpq.mdl***. *The results are shown together with the modified design described in problem 15.5.3.*

15.5.3 *The difficulty with the choice* (15.15.11) *is that it leads to a controller $C(s)$ which cancels the plant poles. This can be checked using* (15.15.12) *to compute $T_o(-1)$ and $T_o(-3)$, since they are not equal to* 1, *hence the poles are effectively canceled. Thus, these poles appear as poles in the input sensitivity.*

It is very important that the reader keeps in mind that cancellation refers to pole-zero cancellation between the controller $C(s)$ and the plant $G_o(s)$. This does not necessarily happen when a pole-zero cancellation occurs between $Q(s)$ and $G_o(s)$.

We recall that a (single) plant pole at $s = p$ is ***not*** *canceled if and only if this pole is a zero of the sensitivity, i.e. if $T_o(p) = 1$.*

We then have to choose $Q(s)$ such as $T_o(-1) = T_o(-3) = T_o(0) = 1$. This requires that $Q(s)$ has at least three free parameters. Say we choose

$$Q(s) = K\frac{(s+1)(s+3)(s^2+as+b)}{(s^2+6s+16)(s+5)(s+6)} \tag{15.15.13}$$

Then, K, a and b are computed to satisfy

$$G_o(-1)Q(-1) = 1 \tag{15.15.14}$$
$$G_o(-3)Q(-3) = 1 \tag{15.15.15}$$
$$G_o(0)Q(0) = 1 \tag{15.15.16}$$

This leads to $K = 32.69$, $a = 5.28$ and $b = 7.34$ and to

$$T_o(s) = 65.38\frac{(s^2+5.28s+7.34)(s+1)(s+3)}{(s+5)(s+6)(s^2+6s+16)} \tag{15.15.17}$$

Figure 15.1 shows the disturbance compensation using the original design (with plant pole cancellation) (15.15.11) *and the modified design* (15.15.13). *For this simulation a zero reference was used and the disturbance was generated by filtering a wide band noise through a low-pass filter with a bandwidth equal to* 4 $[rad/s]$ *(as specified),*

We can observe that, by avoiding cancellation, better disturbance rejection is achieved.

To check whether the modified design changes the reference tracking performance, another simulation is run using a step reference. The results are shown in figure 15.2.

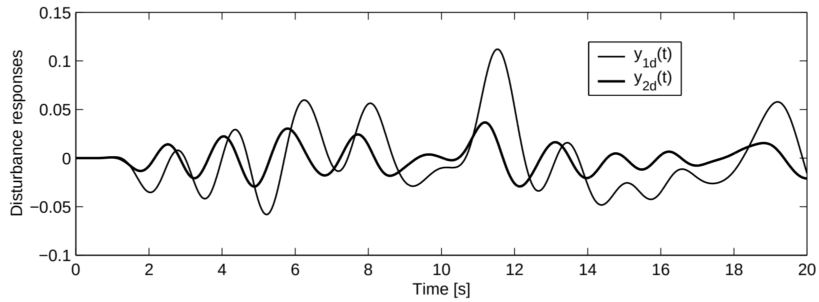

Figure 15.1. Disturbance compensation with (y_{1r}) and without (y_{2r}) plant cancellation

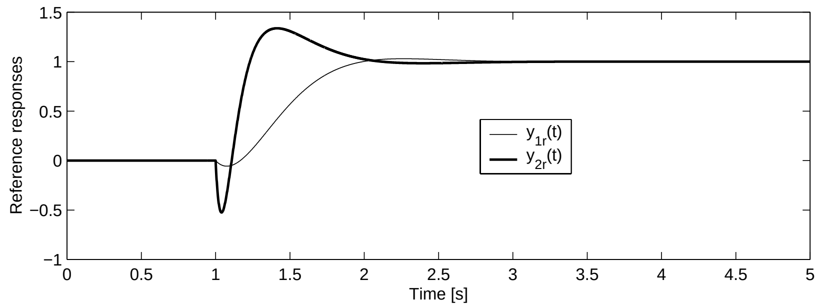

Figure 15.2. Reference tracking with (y_{1r}) and without (y_{2r}) plant cancellation

> *We can now observe that the modified design yields a smaller rise time. However we can also see that larger undershoot and overshoot appear. These negative features are the result of unavoidable trade-offs (see subsection §8.6.5, in the book). On the one hand, we have made the closed loop natural modes faster, this yields increased undershoot (see equation (8.6.28) in the book). On the other hand, since the plant poles are not canceled, they appear as zeros in the sensitivity $S_o(s)$, one of these zeros is at $s = -1$. This produces overshoot that increases as the closed loop (see equation (8.6.27) in the book).*

Problem 15.6. *For $\tau = 0.05$ the loop behaves satisfactorily. This is due to the fact that the modeling error is small, at least in the frequency band of the closed loop. The loop performance is shown in Figure 15.3.*

When $\tau = 0.2$ the control loop becomes unstable. To stabilize the loop, the Q controller transfer function is modified by adding a pole at $s = -4$, this is in agreement with the specified bandwidth. The gain is also modified such that we can still ensure $T_o(0) = 1$. The results are shown in Figure 15.4.