EECE 460: Control System Design
Midterm #3

Dr. Guy Dumont

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This midterm is closed-note, closed book, and only non-graphical calculators are allowed. A one-sided, one-page cheat-sheet is allowed and *must be handed in with your midterm.* For full credit, show all your work.

SOLUTIONS

Student Name Student ID #

Problem #	Actual points	Possible points
1		35
2		35
3		30
Total:		100

1 Problem 1 (35 points)

Consider the system

$$G_0(s) = \begin{bmatrix} \frac{2}{s+1} & \frac{3}{s+2} \\ \\ \frac{1}{s+1} & \frac{1}{s+1} \end{bmatrix}$$

1. (15 points) Show that this system is nonminimum phase

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To find the zeros, need to solve $\det[G_0(s)] = 0 = \frac{2}{(s+1)^2} - \frac{3}{(s+1)(s+2)} = 0.$

$2(s+2) - 3(s+1) = 0 \quad -s+1 = 0 \implies \boxed{s=1}$

RHP zero

2. (10 points) Compute the RGA for the system in question 1. What pairing does it advocate?

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for a 2×2 system,

$$\Lambda = \begin{bmatrix} \lambda & 1-\lambda \\ 1-\lambda & \lambda \end{bmatrix} \quad \text{with} \quad \lambda = \frac{1}{1 - \frac{K_{12}K_{21}}{K_{11}K_{22}}}$$

i.e.

$$\lambda = \frac{1}{1 - \frac{1.5\times 1}{2\times 1.}} = \frac{1}{1-0.75} = 4$$

$$RGA = \begin{bmatrix} 4 & -3 \\ -3 & 4 \end{bmatrix}$$

This advocates the pairing.

$u_1 \to y_1$

$u_2 \to y_2$.

3. (10 points) Consider the singular value plot for the system in question 1. What do they represent? What does they tell you about this system?

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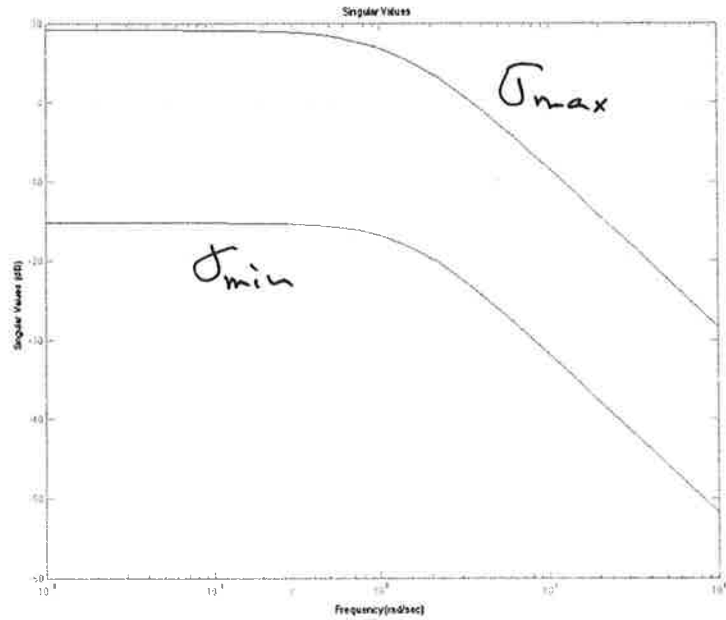

- These show the minimum & maximum gains of the system as f^n of frequency.
- Here $\frac{\sigma_{max}}{\sigma_{min}} \sim 25\,dB$ over the entire frequency means this system has strong directionality and will be difficult to control.

2 Problem 2 (35 points)

Consider the system

$$G_0(s) = \begin{bmatrix} \frac{5e^{-5s}}{4s+1} & \frac{2e^{-4s}}{8s+1} \\ \\ \frac{3e^{-3s}}{12s+1} & \frac{6e^{-3s}}{10s+1} \end{bmatrix}$$

1. (10 points) Derive static decouplers for this system

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For static decoupling consider

$$G_0(0) = \begin{bmatrix} 5 & 2 \\ 3 & 6 \end{bmatrix} = \begin{bmatrix} K_{11} & K_{12} \\ K_{21} & K_{22} \end{bmatrix}.$$

$$Y_1 = K_{11} U_1 + K_{12} U_2$$

Decoupler (1) $\quad K_{11} D_{12} U_2 + K_{12} U_2 = 0$

$$\Rightarrow \quad D_{12} = -\frac{K_{12}}{K_{11}} = \frac{-2}{5} = \boxed{-0.4}$$

$$Y_2 = K_{21} U_1 + K_{22} U_2$$

Decoupler (2) $\quad K_{21} U_1 + K_{22} D_{21} U_1 = 0.$

$$\Rightarrow \quad D_{21} = -\frac{K_{21}}{K_{22}} = \frac{-3}{6} = \boxed{-0.5}$$

2. (15 points) Show that the ideal dynamic decouplers for this system are described by:

$$D_{21}(s) = -\frac{0.5(10s+1)}{12s+1} \qquad \text{and} \qquad D_{12}(s) = -\frac{0.4(4s+1)e^{s}}{8s+1}$$

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Decoupler (1).

$$D_{12} = -\frac{G_{12}}{G_{11}} = \frac{\dfrac{-2e^{-4s}}{(8s+1)}}{\dfrac{5e^{-5s}}{4s+1}} = \boxed{-\frac{0.4(4s+1)e^{s}}{8s+1}}$$

$$D_{21} = -\frac{G_{21}}{G_{22}} = -\frac{\dfrac{3e^{-3s}}{(12s+1)}}{\dfrac{6e^{-3s}}{(10s+1)}} = \boxed{-0.5.\frac{(10s+1)}{12s+1}}$$

3. (10 points) One of the two dynamic decouplers is problematic. Why? How can you reasonably approximate it?

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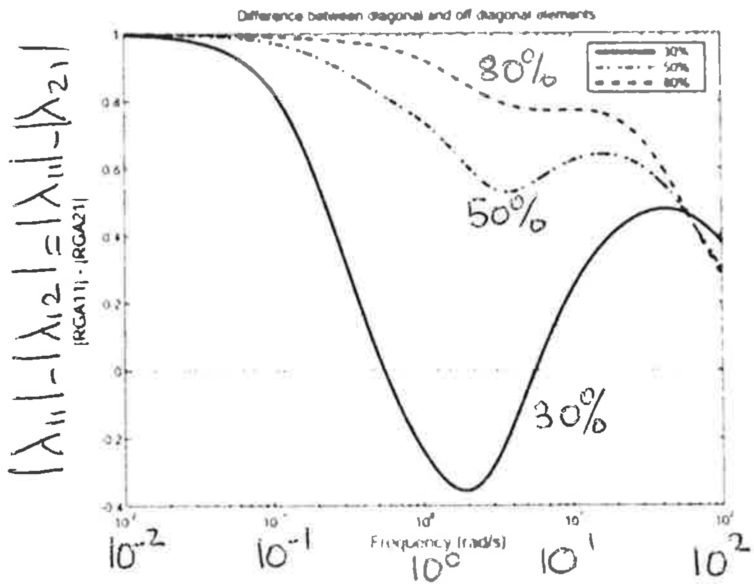
D_{12} include e^{s}, i.e. a non-causal term so cannot be implemented.

Reasonable to approximate it simply as

$$\boxed{D_{12} = -\frac{0.4(4s+1)}{8s+1}}$$

3 Problem 3

The figure below represents the quantity $|\lambda_{11}(s)| - |\lambda_{12}(s)|$ computed from the frequency-dependent RGA $\Lambda(s)$ for a fuel cell for three different power points (30%, 50% and 80%).



1. (15 points) For which power point is this most difficult to control? Why?

Most difficult at 30% power where $|\lambda_{11}| - |\lambda_{12}|$ actually changes sign in the mid-frequency range. At 80%, the system is easiest to control particularly at low frequency.

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2. (15 points) Why can you use a decentralized controller at the high power and expect a reasonable performance for a closed-loop bandwidth of 1 rad/s and yet you should not use that controller at the low power point?

While decentralized control makes sense at high power and low frequencies < 0.1 rad/s where Λ is close to diagonal, the change in sign and rapid drop in $|\lambda_{11}| - |\lambda_{12}|$ in the 0.1 – 10 rad/s at low power makes decentralized control unsuitable at that regime