Problem 2 (30 points)

Consider the following system:

$$G_0(s) = \frac{1}{s^2 + 6s + 5}\begin{bmatrix} s+2 & 3 \\ -(s+1) & (s+4) \end{bmatrix}$$

a) Compute the system's poles and zeros

Poles $s^2+6s+5=0 \Longrightarrow s=-1, -5$

Zeros $\det G(s)=0$

$\det G(s) = \dfrac{(s+2)(s+4)+3(s+1)}{(s^2+6s+5)^2}$

$\Longrightarrow (s+2)(s+4)+3(s+1)=0$

$s^2+6s+8+3s+3=0$

$s^2+9s+11=0$

$s = -\dfrac{9 \pm \sqrt{37}}{2} = -7.54, -1.46$

Poles and zeros

b) Compute the system's relative gain array. What does it represent? On the basis of the RGA, what input-output pairing would you advocate?

$$G(0) = \begin{bmatrix} 2/5 & 3/5 \\ -1/5 & 4/5 \end{bmatrix} = \frac{1}{5}\begin{bmatrix} 2 & 3 \\ -1 & 4 \end{bmatrix}$$

$$\lambda = \frac{1}{1 - \frac{g_{12}g_{21}}{g_{11}g_{22}}} = \frac{1}{1 + \frac{3}{8}} = \frac{8}{11}$$

$$\Lambda = \begin{bmatrix} \lambda & 1-\lambda \\ 1-\lambda & \lambda \end{bmatrix} = \begin{bmatrix} 8/11 & 3/11 \\ 3/11 & 8/11 \end{bmatrix}$$

This suggests the following pairings:

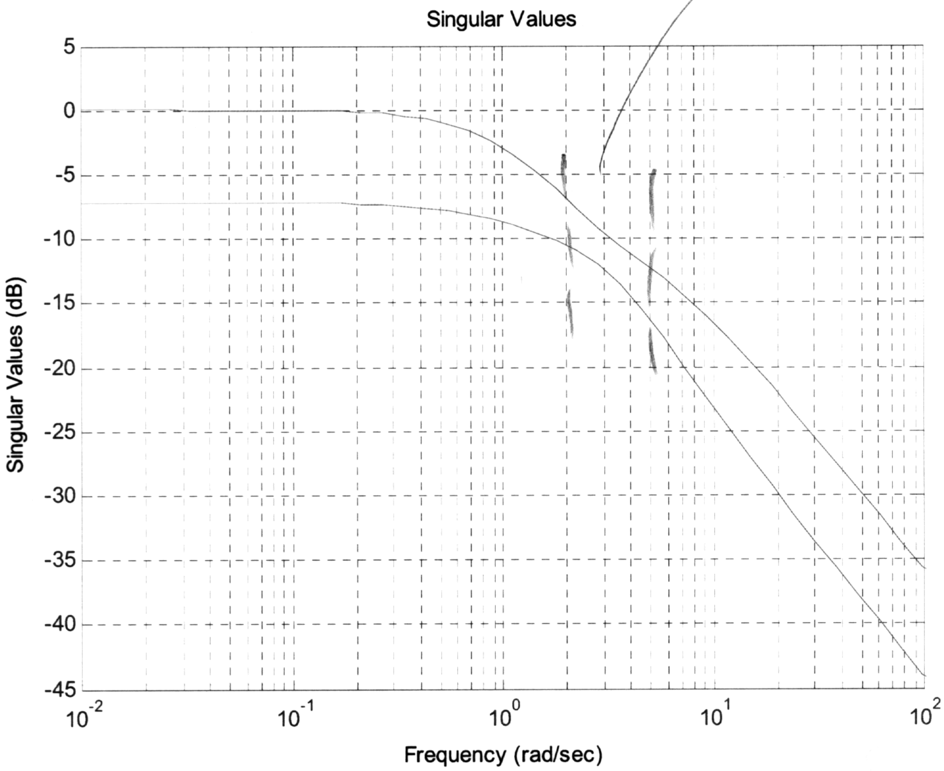
$u_1 \longrightarrow y_1$

$u_2 \longrightarrow y_2$

RGA

c) The singular values for this system are plotted below. What do they tell you about this system?

Also easier to control.



$\bar{\sigma}$ & $\underline{\sigma}$ give max & min gains of system

when $\frac{\bar{\sigma}}{\underline{\sigma}}$ is large, system is likely to be difficult to control. With a ratio $\frac{\bar{\sigma}}{\underline{\sigma}} \simeq 2.5$ this system presents a bit of a challenge, but nothing unsurmountable

Problem 3 (30 points)

Consider the following system:

$$G_0(s) = \frac{1}{s+5}\begin{bmatrix} e^{-s} & 0 \\ e^{-2s} & 5 \end{bmatrix}$$

a) Compute the system's RGA. Is the result surprising?

$$G(0) = \frac{1}{5}\begin{bmatrix} 1 & 0 \\ 1 & 5 \end{bmatrix} \qquad \lambda = \frac{1}{1 - \frac{g_{12}g_{21}}{g_{11}g_{22}}} = 1.$$

$$\Lambda = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

Not surprising given the system has a triangular structure, i.e. one-way coupling.

RGA

b) Design 2 decentralised PI controllers

Decentralized control.

$$G_d(s) = \begin{bmatrix} \frac{e^{-s}}{s+5} & 0 \\ 0 & \frac{5}{s+5} \end{bmatrix}$$

① $\frac{e^{-s}}{s+5}$ λ-tuning.

$$G_1 = \frac{\frac{1}{5}}{\frac{s}{5}+1} e^{-s}$$

$K_p = 0.2 \quad T = 0.2 \quad L = 1.$

$$\boxed{C_1(s) = \frac{1+0.2s}{0.2(\lambda+1)s}}$$

② $\frac{5}{s+5} = \frac{1}{0.2s+1}$

$K_p = 1 \quad T = 0.2, \quad L = 0.$

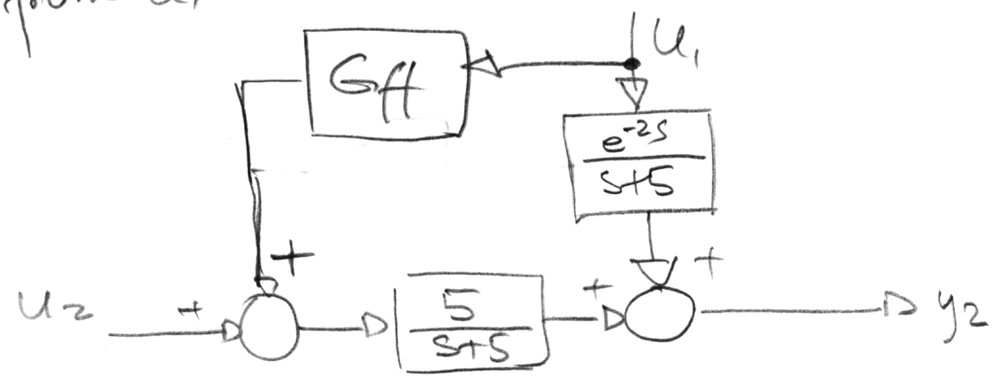
$$\boxed{C_2(s) = \frac{1+0.2s}{\lambda s}}$$

or could use pole-placement.

PI controllers

c) Alter your design accounting for the interaction

Can use feedforward to decouple y_2 from u_1



$$G_{ff} \cdot \frac{5}{s+5} + \frac{e^{-2s}}{s+5} = 0 .$$

$$\boxed{G_{ff} = -0.2\, e^{-2s}}$$

Controllers